

Global coverage 24hr per day from Inclined Circular Orbits

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April 2008

Abstract

By making use of a class of Inclined Orbits (the MEYHO Super-geosynchronous Circular Orbits MSpCOs) for a network of s identical satellites that follow the same ground track with equal time spacing on it, it is possible to provide global coverage 24hr per day, where the s satellites are not placed over the equator in the Geostationary Satellite Orbit (GSO), but enter and exit a predefined small number N of geostationary loops which are widely separated in longitude and latitude and have a common selected latitude north and south of the equator. There is 1 satellite active in all N geostationary loops 24 hr per day, with each satellite not transmitting (i.e. inactive) when outside these loops. The loops, which are portions of the satellite network common ground track, are located in space at a common height above Earth, have wide angular separation from the GSO, and each other, enabling such a network to share the same frequencies among the N loops and with GSO networks through appropriate coordination procedures.

1. Introduction:

1.1 MEYHO Super-geosynchronous Circular Orbits (MSpCOs) with discontinuously operating satellites

By making use of a class of Inclined Orbits (the MEYHO Super-geosynchronous Circular Orbits MSpCOs) for a network of s identical satellites that follow the same ground track with equal time spacing on it, it is possible to provide global coverage 24hr per day, where the s satellites are not placed over the equator in the Geostationary Satellite Orbit (GSO), but enter and exit a predefined small number N of geostationary loops which are widely separated in longitude and latitude and have a common selected latitude north and south of the equator. There is 1 satellite active in all N geostationary loops 24 hr per day, with each satellite not transmitting (i.e. inactive) when outside these loops. The loops, which are portions of the satellite network common ground track, are located in space at a common height above Earth, have wide angular separation from the GSO, and each other, enabling such a network to share the same frequencies among the N loops and with GSO networks through appropriate coordination procedures.

With at least $N=4$ geostationary loops, global coverage $gc(1)$ is made possible 24hr per day at minimum elevation angle ϵ . Being circular, the orbits do not precess, the satellites are at constant height above the Earth, and there is no daily crossing the Van Allan associated with many Highly-inclined Elliptical Orbits (HEOs). Global connectivity may be realized either through inter-satellite links between active satellites in the N different geostationary loops, or making use of gateway stations located in gateway areas where more than 1 geostationary loop is visible at minimum elevation angle ϵ .

The corresponding earth station antenna has either tracking facilities or sufficiently wide beam width to cover an active satellite when it is in a particular geostationary loop.

Inclined Orbits with period $p=(n+1)/n \cdot 24$ hr (the MEYHO Super-geosynchronous Circular Orbits), provide global coverage $gc(1)$ such that from any location on earth at least 1 of the N geostationary loops is visible at minimum elevation angle ϵ , but require a number s of identical satellites in the network, more than was the case for global non-polar coverage $gnpc(1)$ using the GSO. The size of the geostationary loop is measured in terms of Equivalent Angular Dimension (EAD) and its length T hr. The more satellites in the network, the smaller the size of the geostationary loop. (For a satellite in the GSO, the geostationary loop is a point above the equator with EAD dimensions dependent of degree of station keeping).

(Other Circular Inclined Super-geosynchronous Orbits with period $p=q/r \cdot 24$ hr, $q>r+1$ are higher than MSpCOs and thus less suitable for providing global coverage at minimum elevation angle ϵ .)

With $N=4$ geostationary loops, there will be on Earth $N(N-1)/2=6$ gateway areas (where at least 2 geostationary loops of the six combinations are visible,) with no more than 2 space loop hops required for a link between any 2 locations on Earth. Alternatively there are 6 inter-loop connections that may be established between active satellites in the 4 geostationary loops, where no more than 1 such connection and 1 space loop hop is required for a link between any 2 locations on Earth.

With $N=6$ geostationary loops, it is possible to provide global coverage with greater communications capacity than with only $N=4$ geostationary loops for the same minimum elevation angle ϵ , but this requires more gateway areas/inter-loop links to provide global connectivity. There will be at least $(N-2)(N-3)=12$ gateway areas (where 2 geostationary loops are visible, with no more than 3 space loop hops required for a link between any 2 locations on Earth.

Alternatively there are $(N-2)(N-3)=12$ inter-loop connections that may be established between the 6 geostationary loops where no more than 2 such connections and 1 space loop hop are required for a link between any 2 locations on Earth.

With $N=8$ geostationary loops, it is possible to provide global coverage $gc(2)$, such that from any location on earth at least 2 geostationary loops are visible at minimum elevation angle ϵ .

We examine in Appendix 1 global coverage $gc(r)$ of a non-rotating sphere radius r_a from a number N of fixed points at the same minimum constant height $h(\epsilon)$ above the surface of the sphere, so that from any location on the surface of the sphere it is possible to view at least $r < N$ points above elevation angle ϵ . The results may then be applied to global coverage of a rotating sphere such as the Earth with 24 hr period, where the N points become N geostationary loops. Satellites operate only in these geostationary loops, and are switched off in other parts of their inclined circular orbits.

2. Geostationary Loops formed using MEYHO Super-geosynchronous Circular Orbits (MSpCOs)

2.1 Ground track of satellite orbiting the Earth

The ground track of a satellite in a Circular Inclined Orbit (CIO) with period $p=m/n \cdot 24$ hr, where m, n are integers with no common factor, around the rotating sphere Earth, is repetitive every $m \cdot 24$ hr with $2n$ separate peaks around the latitudes $\pm i$ corresponding to the inclination angle i of the CIO w.r.t the Earth's equator. The ground track **resembles** a convoluted sinusoid over the range of longitudes $\lambda = \pm \pi$. For the special cases of period $p=(n+1)/n \cdot 24$ hr, [MEYHO Super-geosynchronous Circular Orbit (MSpCO)], and period $p=(n-1)/n \cdot 24$ hr [MEYHO Sub-geosynchronous Circular Orbit (MSbCO)], the ground track has a simple non-convoluted sinusoid waveform.

The non-convoluted sinusoid waveform of the MSpCO develops $2n=N$ separate loops of duration $T(i)$ hr around latitudes $\pm i$, with longitudinal separation π/n between adjacent loops. In this way this orbit provides N geostationary loops. The satellite will be switched on only inside these N loops, which are widely separated in angle from each other and from the Geostationary Satellite Orbit (GSO). The MSpCO being a circular orbit, the satellite in it maintains a constant height $h(\epsilon)$ above the surface of the Earth.

From any location on Earth, at least r of the N geostationary loops will be visible above minimum elevation angle ϵ . The corresponding earth station antenna beam pointing at that (those) geostationary loop(s) may be wide enough to cover the satellite while it is in the loop, or may track it during its passage through the loop. The height of the circular orbit $h(\epsilon)$ is related to its period, p which then determines the above minimum elevation angle ϵ from any earth station.

These N geostationary loops correspond to the N fixed points around a non-rotating sphere discussed in Appendix A1.2.2($N=4$); A1.3.2($N=6$); A1.4.2, 1.4.3($N=8$). The inclination angle i of the MSpCO corresponds to the latitude ϕ for the non-rotating sphere in the Appendix. The latitude ϕ determined the coverage angle ψ , for these configurations of N points to provide global coverage (Tables A1.1.2[$N=4$]; A1.3.2[$N=6$]; A1.4.2, 1.4.3[$N=8$]). The height $h(\epsilon)$ of the MSpCO above the Earth's radius $r_a=6380$ km being a function of the orbit period $p=(n+1)/n \cdot 24$ hr will provide global coverage from the geostationary loop at minimum elevation angle ϵ , where from equations derived in the Appendix,

$$\tan \epsilon = [\cos \psi - r_a / \{h(\epsilon) + r_a\}] / \sin \psi \quad \text{A1.1}$$

$$\psi = \cos^{-1}[\sin i / \{1 + 4 \tan^2(i)\}] \quad [N=4, \text{gc}(1)] \quad \text{A1.1.2.3}$$

$$\psi = i + \tan^{-1}[1/4 \tan i] \quad [N=6, \text{gc}(1)] \quad \text{A1.3.2.4}$$

$$\psi = \cos^{-1}[\sin i / \{1 + 8 \tan^2(i)\}] \quad [N=8, \text{gc}(2)] \quad \text{A1.4.2.5}$$

$$\psi = i + \tan^{-1}[1/2 \tan i] \quad [N=8, \text{gc}(2)] \quad \text{A1.4.3.6}$$

The size of the loop of duration $T(i)$ hr is measured in terms of **Equivalent Angular Dimension (EAD)**, (Table 2.1), where

$$\text{EAD} = \sqrt{(\Delta \text{latitude}) \cdot (\Delta \text{longitude}) \cdot \cos(\text{mean latitude of loop})} \quad \text{2.1.1}$$

$\Delta \text{latitude}$ = difference in latitude of the loop

$\Delta \text{longitude}$ = difference in longitude of the loop

There is a minimum value of inclination angle i_m ($T(i)=0^{\circ}0$) below which no loop occurs in the geostationary ground track of the MSpCO.

T(i)hr			0				4				6				8			
(n+1)/n	h km	ra/(h+ra)	i-m?	i?	ψ ?	ϵ ?	EAD?	i?	ψ ?	ϵ ?	EAD?	i?	ψ ?	ϵ ?	EAD?			
2/1*	60553	.0953	60	62.4	*	*	2	65.7	*	*	5	70.6	*	*	11			
3/2	48873	.1155	48	50.9	73.0	10.5	2	54.7	74.2	9.25	5.5	61.0	76.5	6.9	13			
4/3	44700	.1249	41	44.1	58.6	24.9	2	48.0	60.7	22.7	5	54.8	64.8	19.0	13			
5/4	42549	.1304	37	39.5	75.5	7.1	1.5	43.4	76.1	6.5	5	50.3	77.5	5.1	12			

* no global coverage

Table 2.1

2.2 Global coverage $gc(r)$ of the Earth 24 hr per day using MSpCOs

Global coverage $gc(r)$ of the Earth 24 hr per day would require 1 satellite to be inside each loop of the MSpCOs common ground track at all times, so that the satellite network would contain $s=(n+1).24/T(i)$ identical satellites with equal time spacing $T(i)$ hr between the satellites tracing that same ground track. Then as 1 satellite enters a geostationary loop, another satellite exits that same loop at the same point in space.

A satellite $Sq[an(q),ma(q)]$, $q=1,s$ in a particular inclined orbit may be described in terms of its ascending node (an) and mean anomaly (ma). For s identical satellites to trace the same ground track with equal time spacing $T(i)$ hr between them, there is a relationship between the parameters s , $p=m/n.24$ hr., $an(q),ma(q)$, where the corresponding orbit for each satellite has the same inclination angle i and the same period p . The mean anomaly difference Δma between adjacent satellites is

$$ma(q+1)=ma(q)+\Delta ma, \text{ where } \Delta ma= n.2\pi/s= 2\pi T(i)/p \quad 2.2.1$$

The approximate angular separation $\gamma_{gs}= i$ of the MSpCO geostationary loops from the GSO is large as are the approximate angular separations γ_{ss1} , γ_{ss2} between adjacent loops where (Table 2.2.1)

γ_{ss1}	γ_{ss2}	N
$\pi-2i$	$\cos^{-1}[-\sin^2(i)]$	4 (A1.1.2)
$\cos^{-1}[\sin^2(i)-0.5\cos^2(i)]$	$\cos^{-1}[-\sin^2(i)+ 0.5\cos^2(i)]$	6 (A1.3.2)
$\cos^{-1}[\sin^2(i)]$	$\cos^{-1}[-\sin^2(i)+0.5\cos^2(i)]$	8 (A1.4.2)
$\cos^{-1}[\sin^2(i)]$	$2i$	8 (A1.4.3)

Table 2.2.1

For $Sq[an(q),ma(q)]$, $q=1,s$ identical satellites equally time spaced $T(i)$ hr in MSpCOs having the same ground track with N geostationary loops of length $T(i)$ hr, some examples are given below (Table 2.2.2):

gc(r)	N	n	T(i) hr	s	Δma°	$i^\circ=\gamma_{\text{gs}}^\circ$	$\gamma_{\text{ss1}}^\circ$	$\gamma_{\text{ss2}}^\circ$	h km	Fig.
1	4	2	4	18	40	51	78	127	48873	2.2.2
1	6	3	6	16	67.5	48	71	109	44700	2.2.3
2	8	4	8	15	96	50	54	107	42549	2.2.4
2	8	2x4	8	2x9=18	80	61	40	122	48873	2.2.5
2	8	4x1	8	4x6=24	60	71	27	142	60553	2.2.6

S1(0,0) S2(0,40) S3(0,80) S4(0,120) S5(0,160) S6(0,200) S7(0,240) S8(0,280) S9(0,320)
S10(180,0) S11(180,40) S12(180,80) S13(180,120) S14(180,160) S15(180,200) S16(180,240)
S17(180,280) **S18**(180,320) Fig.2.2.2

S1(0,0) S2(0,67.5) S3(0,135) S4(0,202.5) S5(0,270) S6(0,337.5) S7(240,45) S8(240,112.5)
S9(240,180) S10(240,247.5) S11(240,315) S12(120,22.5) S13(120,90) S14(120,157.5)
S15(120,225) **S16**(120,292.5) Fig.2.2.3

S1(0,0) S2(0,96) S3(0,192) S4(0,288) S5(270,24) S6(270,120) S7(270,216) S8(270,312) S9(180,48)
S10(180,144) S11(180,240) S12(180,336) S13(90,72) S14(90,168) **S15**(90,264) Fig.2.2.4

S1(0,0) S2(0,80) S3(0,160) S4(0,240) S5(0,320) S6(180,40) S7(180,120) S8(180,200) S9(180,280)
S10(90,0) S11(90,80) S12(90,160) S13(90,240) S14(90,320) S15(270,40) S16(270,120)
S17(270,200) **S18**(270,280) Fig.2.2.5

S1(0,0) S2(0,60) S3(0,120) S4(0,180) S5(0,240) S6(0,300) S7(90,0) S8(90,60) S9(90,120)
S10(90,180) S11(90,240) S12(90,300) S13(180,0) S14(180,60) S15(180,120) S16(180,180)
S17(180,240) S18(180,300) S19(270,0) S20(270,60) S21(270,120) S22(270,180) S23(270,240)
S24(270,300) Fig.2.2.6

3. Conclusion

Global coverage 24 hr per day cannot be achieved from the Geostationary Satellite Orbit (GSO), the Polar regions being excluded. It would require Inclined Circular Orbits (ICOs). At the same time satellites in ICOs must be switched off during part of their trajectory, thereby operating only from geostationary positions (associated with their orbits), which have a wide angular separation from the GSO and each other for efficient orbit spectrum utilization. By selecting the Meyho Super- geosynchronous Circular Orbits (MspCOs), a class of ICOs, identical satellites provide global coverage in frequency bands that can be shared with satellite networks in the GSO. The corresponding earth station antenna beams may either track the operating satellite in its geostationary loop, or be sufficiently wide to cover that loop. There are a number of new applications for satellite networks that call for global coverage rather than the limited coverage available for the GSO.

Appendix

A1. Global coverage of a non-rotating sphere

The non-rotating sphere is considered to have longitude λ and latitude ϕ coordinates.

To provide global coverage $gc(r)$ of a non-rotating sphere at minimum elevation angle ϵ requires at least N fixed points $P_{q=1,N}$ at minimum height ratio $h(\epsilon)/r_a$ where r_a is the radius of the sphere. Then from any location on the sphere at least r such points are visible above elevation angle ϵ where,

$$\tan \epsilon = [\cos \psi - r_a / \{h(\epsilon) + r_a\}] / \sin \psi \quad A 1$$

X, Y are the coordinates of the location on the non-rotating sphere that determine the minimum coverage angle ψ from any point that insures global coverage from the N points at minimum height ratio h/r_a . The angular separation between points is $P_q - P_s = \alpha_{qs}$. The points P_q, P_s may be linked provided $\alpha_{qs}/2 < \psi$. It is not possible to provide global coverage $gc(1)$ with $N < 4$ points.

1 A1.1 N= 4 points, gc(1)

2 A1.1.11 polar point P1 & 3 non-equatorial points P3,P4,P5, at latitude ϕ with equal adjacent longitudinal spacing $2\pi/3$

The $N=4$ fixed points $P_q, q=1, N$ have angular separation $P_1 - P_q = \alpha_{1q}, q=2, N$ & $P_q - P_r = \alpha_{qr}, 1 < q < r < N+1$ and are located at one Pole (P_1) with the remaining 3 points (P_2, P_3, P_4) at latitude ϕ in the other hemisphere, each separated in longitude by $2\pi/3$ (Table A1.1.1).

X is the latitude of location at mid-longitude $Y, \pi/3$ between 2 adjacent points P_2, P_3 at latitude ϕ on the surface of the non-rotating sphere with equal angular separation ψ from Points P_1, P_2, P_3 . ψ is the minimum required coverage angle from any point $P_q, q=1, N$ to provide global coverage $gc(1)$.

$$\tan X = \cos \phi \cdot \cos(\pi/3) / (1 + \sin \phi) \text{ and } \psi = \pi/2 - X > \pi/2 - \phi \quad A1.1.1$$

$$\alpha_{1q} = \pi/2 + \phi, q=2, N \text{ and } \cos(\alpha_{qr}) = \sin^2(\phi) - \cos^2(\pi/3) \cdot \cos^2(\phi), 1 < q < r < N+1 \quad A1.1.2$$

There are 3 locations for longitude Y , corresponding to X latitude.

For $\phi < 19.5$ to achieve global coverage $gc(1)$ requires at least $\psi = \pi/2 - \phi$, (now no longer related to X).

The minimum height ratio h/r_a is given by

$$h/r_a = 1/\cos \psi - 1 \quad A1.1.3$$

All points are able to connect to the remaining $N-1$ points provided $\alpha_{1q}/2$ & $\alpha_{qr}/2 < \psi$.

$\phi?$	10	19	19.5	20	30	40	50	60	70	80
$X?$	-	-	19.5	19.3	16.1	13.1	10.3	7.63	5.04	2.50
$\psi?$	80	71	70.5	70.7	73.9	76.9	79.7	82.4	85.0	87.5
$(\pi/2 - \phi)?$	80	71	70.5	70	60	50	40	30	20	10
$\alpha_{1q}?$	100	109	109.5	110	120	130	140	150	160	170
$\alpha_{qr}?$	117	109.9	109.5	108.9	97.2	83.1	67.7	51.3	34.5	17.3
h/r_a	4.76	2.07	2	2.03	2.61	3.41	4.59	6.56	10.5	21.9

Table A1.1.1

Optimum choice for the N=4 points for gc(1) is related to their uniform distribution around the non-rotating sphere, with equal adjacent angular separation $\alpha_{1q}=\alpha_{qr}$ corresponding to the lowest value for height ratio h/ra to provide global coverage gc(1), i.e.

$$\varphi=19.5, \psi=\pi/2 - \varphi =70.5, \alpha_{1q}=\alpha_{qr}=\pi - \psi=109.5, h/ra=2 \quad \text{A 1.1.1.4}$$

However these selections of N=4 points do not provide a suitable basis for global coverage gc(1) of the rotating Earth 24 hr per day since it is not practical to convert these points into N geostationary positions (loops).

1 A1.1.2 N=4 points equally adjacently spaced $2\pi/N$ in longitude on a N/2 cycle sinusoid

Corresponding to a N/2 cycle sinusoid over longitude, the N=4 fixed points, $P_q=1, N$ may be located respectively above latitudes $\pm\varphi$, with $2\pi/N$ longitudinal spacing between adjacent points, yielding angular spacing between the points (Table A1.1.2),

$$\alpha_{12}=\alpha_{34}=\pi-2\varphi$$

A1.1.2.1

$$\alpha_{13}=\alpha_{14}=\alpha_{23}=\alpha_{24}=\cos^{-1}[-\sin^2(\varphi)] \quad \text{A1.1.2.2}$$

The minimum coverage angle ψ from any of the above points P_q to provide gc(1) is derived from $\pm X$, the latitude of location at mid-longitude Y, $2\pi/N$ between 2 adjacent points P_1, P_2 at latitude φ on the surface of the non-rotating sphere yielding equal angular separation ψ from Points P_1, P_2, P_3 .

There are 4 locations for longitude Y corresponding to latitudes $\pm X$ respectively.

$$\cos \psi = \sin \varphi \cdot \sin X = \cos(\varphi + X) = \sin \varphi / [1 + 4 \tan^2(\varphi)] \quad \text{A1.1.2.3}$$

φ°	10	20	30	35.25	40	50	60	70	80
α_{12}°	160	140	120	109.5	100	80	60	40	20
α_{13}°	91.7	96.7	104.5	109.5	114.5	125.9	138.6	152	166
ψ°	80.6	73.9	70.9	70.5	70.8	72.8	76.1	80.3	85.0
X°	70.6	53.9	40.9	35.25	30.8	22.8	16.1	10.3	5
h/ra	5.12	2.61	2.06	2.0	2.04	2.38	3.16	4.94	10.47

Table A1.1.2

For $\varphi=35.25$, the N=4 points are uniformly spaced around the non-rotating sphere ($\alpha_{11}=\alpha_{12}$), and correspond to the uniform distribution of N=4 points given above in paragraph A1.1.1.

These distributions of N=4 points provide the basis for global coverage gc(1) of the rotating Earth 24 hr per day where these N points are converted into N geostationary positions (loops) at latitudes $\pm\varphi$ with adjacent longitudinal spacing $2\pi/N$ using Meyho Super-geosynchronous Circular Orbits.

A1.2 N= 5 points, gc(1)

A1.2.1 2 polar points P1,P2 & 3 equatorial points P3,P4,P5 with equal longitudinal spacing $2\pi/3$

Coverage of a non-rotating sphere from N=5 fixed points $P_q, q=1, N$ will have the points located at the same height ratio h/ra above the non-rotating sphere one at each Pole (P1,P2) with the remaining $P_q, q=3, N$ in the equatorial plane, with adjacent longitudinal spacing $2\pi/3$.

The angular separation between points P_q, P_r is α_{qr} where

$$\alpha_{12}=\pi, \alpha_{1q}=\alpha_{2q}=\pi/2, q=3, N \quad \alpha_{34}=\alpha_{45}=\alpha_{53}=2\pi/3 \quad \text{A1.2.1.1}$$

The minimum coverage angle ψ from any of the above points to provide gc(1) is derived from +X, the latitude of location at mid-longitude Y, $\pi/3$ between 2 adjacent points P3,P4 above the equator of the non-rotating sphere yielding equal angular separation ψ from Points P1,P3,P4. ψ is the required coverage angle from any point $P_q, q=1, N$ to provide global coverage gc(1)

$$\psi = \pi/2 - X = \cos^{-1}[\cos X \cdot \cos \pi/3] \quad \text{i.e. } \psi^\circ = 63.4 \quad h/ra = 1.236 \quad \text{A1.2.1.2}$$

There are 3 locations for longitude Y corresponding to latitudes +/-X respectively..

All equatorial points $P_q, q=3, N$ are able to connect to N-1 remaining points, while the polar points P1,P2 are able to connect to N-2 remaining points.

1 This polar configuration may be transformed where the Points $P_q, q=1, N$ have the same angular separation α_{qr} as above. P1, P2, P3 are located on the equator at longitudes $\pi/2, 3\pi/2, 0$ and P4,P5 at latitudes +/- $\pi/3$, longitude π , respectively.

However these selections of N=5 points do not provide a suitable basis for global coverage gc(1) of the rotating Earth 24 hr per day since it is not practical to convert these points into N geostationary positions (loops).

1 A1.2.2 N=5 points equally adjacently spaced $2\pi/N$ in longitude on a (N-1)/2 cycle sinusoid

Coverage of a non-rotating sphere based on (N-1)/2 cycle sinusoidal distribution of the fixed points over longitude has the N=5 points $P_q(\lambda_q, \phi_q), q=1, N$ at the same height ratio h/ra above the non-rotating sphere located with longitude, latitude coordinates $\lambda_q = (q-1)2\pi/N, \phi_q = \phi \cos \lambda_q (N-1)/2$ respectively, having angular separation

$$\alpha_{12}=\alpha_{15}=\cos^{-1}[-\sin \phi \cdot \sin 0.809\phi + 0.309 \cdot \cos \phi \cdot \cos 0.809\phi] \quad \text{A1.2.2.1}$$

$$\alpha_{23}=\alpha_{45}=\cos^{-1}[-\sin 0.809\phi \cdot \sin 0.309\phi + 0.309 \cdot \cos 0.309\phi \cdot \cos 0.809\phi] \quad \text{A1.2.2.2}$$

$$\alpha_{34} = \cos^{-1}[\sin^2(0.309\phi) + 0.309 \cdot \cos^2(0.309\phi)]$$

A1.2.2.3

$$\alpha_{13}=\alpha_{14}=\cos^{-1}[\sin\phi.\sin 0.309\phi -0.809.\cos\phi.\cos 0.309\phi] \quad \text{A1.2.2.4}$$

$$\alpha_{24}=\alpha_{35}=\cos^{-1}[-\sin 0.809\phi.\sin 0.309\phi -0.809.\cos 0.309\phi \cos 0.809\phi]$$

1.2.2.5

$$\alpha_{25} =\cos^{-1}[\sin^2 (0.809\phi) -0.809.\cos^2 (0.809\phi)] \quad \text{1.2.2.6}$$

X3 is the latitude of location, longitude 0, on the non-rotating sphere yielding equal angular separation ψ_3 from Points P1,P2,P5, and angular separation $\psi_a > \psi_3$ from location X3,0 to Points P3,P4 where,

$$\tan X_3 = (\cos\phi -0.309.\cos 0.809\phi)/(\sin\phi +\sin 0.809\phi) \quad \text{A1.2.2.7}$$

$$\psi_3 = \phi + X_3 \quad \text{A1.2.2.8}$$

$$\psi_a = \cos^{-1}[-\sin X_3.\sin 0.309\phi -0.809\cos X_3 \cos 0.309\phi] \quad \text{A1.2.2.9}$$

while X4 is the latitude of location, longitude π , on the non-rotating sphere yielding equal angular separation ψ_4 from Points P2,P3,P4, P5 and angular separation $\psi_b > \psi_4$ from location X4, π to Point P1 where,

$$\tan X_4 = (0.309.\cos 0.809\phi +0.809.\cos 0.309\phi)/(\sin 0.309\phi +\sin 0.809\phi) \quad \text{A1.2.2.10}$$

$$\psi_4 = \sin X_4.\sin 0.809\phi -0.309.\cos X_4.\cos 0.809\phi \quad \text{A1.2.2.11}$$

$$\psi_b = \pi -(\phi -X_4) \quad \text{A1.2.2.12}$$

All points are able to connect to the remaining N-1 points provided $\alpha_{qr}/2 < (\psi_3 \text{ or } \psi_4)$ which ever is larger since the larger value determines the minimum value of h/ra.

The optimum value for latitude ϕ for global coverage $gc(1)$ occurs when $\psi_3 = \psi_4$, i.e. $\phi = 58.5$ (Table A1.2.2), but even this configuration results in a higher value for ψ than in paragraph 1.2.1 above. Neither of these 2 cases constitutes a uniform distribution of N fixed points over the non-rotating sphere.

ϕ°	α_{12}°	α_{23}°	α_{34}°	α_{13}°	α_{24}°	α_{25}°	X_3°	ψ_3°	X_4°	ψ_4°	ψ_b°	ψ_a°	h/ra
50	110	86.9	69.0	107	140	92.7	16.07	66.07	47.96	71.07	178	145	2.08
55	117	89.7	68.4	102	139	85.4	13.08	68.08	45.03	70.12	170	145	1.94
58.5	121	91.7	67.9	97.9	139	80.3	11.16	69.66	43.09	69.56	165	145	1.88
60	123	92.5	67.7	96.2	138	78.1	10.36	70.36	42.31	69.32	162	144	1.98
70	137	98.6	66.2	84.9	136	63.1	5.54	75.54	37.45	68.11	147	142	3.0

Table A 1.2.2

However this selection of N=5 points does not provide a suitable basis for global coverage $gc(1)$ of the rotating Earth 24 hr per day since it is not practical to convert these points into N geostationary positions (loops).

A1.3 N= 6 points, gc(1)

A1.3.1 2 polar points P1,P2 & 4 equatorial points P3,P4,P5, P6 with equal adjacent longitudinal spacing $\pi/2$

Coverage of a non-rotating sphere from N=6 fixed points $P_q, q=1, N$ will yield global coverage $gc(1)$ when the points at the same height ratio h/ra above the non-rotating sphere are located one at each Pole (P1,P2) with the remaining $P_q, q=3, N$ in the equatorial plane, with adjacent longitudinal spacing $\pi/2$. This constitutes a uniform distribution of the points around a non-rotating sphere.

The angular separation between points P_q, P_r is α_{qr} where

$$\alpha_{12}=\alpha_{35}=\alpha_{46}=\pi, \alpha_{1q}=\alpha_{2q}=\alpha_{34}=\alpha_{45}=\alpha_{56}=\alpha_{63}=\pi/2, \quad q=3, N \quad A1.3.1.1$$

The minimum coverage angle ψ from any of the above points to provide $gc(1)$ is derived from

$$\cos\psi = \cos X \cdot \cos\pi/4 = 1/\sqrt{3} \quad \text{i.e. } \psi^\circ = 54.74, h/ra = 0.732 \quad A1.3.1.2$$

There are 4 locations for longitude Y corresponding to +/-X latitudes respectively.

All points are able to connect to N-2 remaining points with equal angular spacing $\pi/2$ for these $N(N-2)/2$ links.

1 This polar configuration may be transformed where the Points $P_q, q=1, N$ have the same angular separation α_{qr} as above. P1,P2 are located on the equator at longitudes $\pi/2, 3\pi/2$ and P3,P4, at latitudes +/- $\pi/4$, longitude 0, and P5,P6 at latitudes +/- $\pi/4$, longitude π respectively.

However this selection of N=6 fixed points does not provide a suitable basis for global coverage $gc(1)$ of the rotating Earth 24 hr per day since it is not practical to convert these points into N geostationary positions (loops).

A1.3.2 N=6 points equally adjacently spaced $2\pi/N$ in longitude on a N/2 cycle sinusoid

The N=6 fixed points, $P_q, q=1, N$ may be located respectively above latitudes +/- ϕ , with $2\pi/N$ longitudinal spacing between adjacent points, yielding angular spacing between the points (Table A1.3.2),

$$\alpha_{12}=\alpha_{23}=\alpha_{31}=\alpha_{45}=\alpha_{56}=\alpha_{64}=\cos^{-1}[\sin^2(\phi)-\cos^2(\phi)\cdot\cos\pi/3] \quad A1.3.2.1$$

$$\alpha_{14}=\alpha_{24}=\alpha_{25}=\alpha_{35}=\alpha_{36}=\alpha_{61}=\pi-\alpha_{12} \quad A1.3.2.2$$

$$\alpha_{15}=\alpha_{26}=\alpha_{34}=\pi \quad A1.3.2.3$$

The minimum coverage angle ψ from any of the above points to provide $gc(1)$ is derived from

$$\psi = \cos^{-1}[\sin\phi \cdot \sin X + \cos\phi \cdot \cos X \cdot \cos\pi/3] = \phi + X = \phi + \tan^{-1}[1/4 \tan\phi] \quad A1.3.2.4$$

There are 4 locations for longitude Y, corresponding to latitudes +/-X respectively for $\phi^\circ > 35.25$.

For $\phi^\circ < 35.25$ to achieve global coverage $gc(1)$ requires at least $\psi = \pi/2 - \phi$, (now no longer related to X).

All points are able to connect to N-2 remaining points provided $\alpha_{qr}/2 < \psi$, i.e. a total of $N(N-2)/2$ links.

ϕ°	10	20	30	35.25	40	50	60	70	80
$\alpha 12^\circ$	117	109	97.2	90	83.1	67.7	51.3	34.5	17.3
$\alpha 14^\circ$	63	71	82.8	90	96.9	112	129	145.5	163
ψ°	80	70	60	54.74	56.6	61.8	68.2	75.2	82.5
X°				19.5	16.6	11.8	8.2	5.2	2.5
h/ra	4.76	1.92	1.0	0.732	0.817	1.116	1.69	2.91	6.66

Table A1.3.2

These distributions of N=6 points provide the basis for global coverage $gc(1)$ of the rotating Earth 24 hr per day where these N points are converted into N geostationary positions (loops) at latitudes $\pm\phi$ with adjacent longitudinal spacing $2\pi/N$ using Meyho Super-geosynchronous Orbits.

$\phi^\circ=35.25$ constitutes a uniform distribution of the N=6 points around a non-rotating sphere equivalent to the case above in paragraph A1.3.1.

A1.4 N=8 points, gc(2)

A1.4.1 2 polar points P1,P2 & 6 non-equatorial points P3,P4,P5,P6,P7,P8 with adjacent longitudinal spacing $\pi/3$ at latitudes $\pm\phi=19.5$

Coverage of a non-rotating sphere from N=8 fixed points $P_q, q=1, N$ will yield global coverage gc(2) when the points at the same height ratio h/ra above the non-rotating sphere are located one at each Pole (P1,P2) with the remaining $P_q, q=3, N$ at latitudes $\pm\phi=19.5$, with adjacent longitudinal spacing $\pi/3$. This constitutes a uniform distribution of the points around a non-rotating sphere.

The angular separation between points P_q, P_r is α_{qr} where

$$\alpha_{12}=\alpha_{37}=\alpha_{48}=\alpha_{56}=\pi,$$

$$\alpha_{13}=\alpha_{14}=\alpha_{15}=\alpha_{26}=\alpha_{27}=\alpha_{28}=\alpha_{36}=\alpha_{38}=\alpha_{46}=\alpha_{47}=\alpha_{57}=\alpha_{58}=\psi=\pi/2-\phi$$

$$\alpha_{16}=\alpha_{17}=\alpha_{18}=\alpha_{23}=\alpha_{24}=\alpha_{25}=\alpha_{34}=\alpha_{35}=\alpha_{45}=\alpha_{67}=\alpha_{68}=\alpha_{78}=\pi-\psi \quad \text{A1.4.1}$$

The minimum coverage angle from any of the above points to provide gc(2) is $\psi^0=70.5$ when the determining location X,Y on the non-rotating sphere is below any of the N=8 points, and the minimum height ratio $h/ra=2$.

All points are able to connect to N-2 remaining points with angular spacing ψ for (N-2)/2 points & angular spacing $\pi-\psi$ for the other (N-2)/2 points.

However this selection of N=8 points does not provide a suitable basis for global coverage gc(2) of the rotating Earth 24 hr per day since it is not practical to convert these points into N geostationary positions (loops).

1 A1.4.2 N=8 points equally adjacently spaced $2\pi/N$ in longitude on a N/2 cycle sinusoid

The N=8 fixed points, $P_q, q=1, N$ may be located respectively above latitudes $\pm\phi$, with $2\pi/N$ longitudinal spacing between adjacent points, yielding angular spacing between the points (Table A1.4.2),

$$\alpha_{12}=\alpha_{23}=\alpha_{34}=\alpha_{41}=\alpha_{56}=\alpha_{67}=\alpha_{78}=\alpha_{85}=\cos^{-1}[\sin^2(\phi)] \quad \text{A1.4.2.1}$$

$$\alpha_{15}=\alpha_{25}=\alpha_{26}=\alpha_{36}=\alpha_{37}=\alpha_{47}=\alpha_{48}=\alpha_{81}=\cos^{-1}[-\sin^2(\phi)+\cos^2(\phi).\cos\pi/4] \quad \text{A1.4.2.2}$$

$$\alpha_{16}=\alpha_{27}=\alpha_{38}=\alpha_{45}=\alpha_{35}=\alpha_{46}=\alpha_{17}=\alpha_{82}=\cos^{-1}[-\sin^2(\phi)-\cos^2(\phi).\cos\pi/4] \quad \text{A1.4.2.3}$$

$$\alpha_{13}=\alpha_{24}=\alpha_{57}=\alpha_{68}=\pi-2\phi \quad \text{A1.4.2.4}$$

The minimum coverage angle ψ from any of the above points to provide gc(2) is derived from

$$\cos\psi = \sin\phi.\sin X = -\sin\phi.\sin X + \cos\phi.\cos X.\cos\pi/4 = \sin\phi/[1+8\tan^2(\phi)] \quad \text{A1.4.2.5}$$

There are 8 locations for longitude Y, corresponding to the longitude of the 8 points at $\pm X$ latitudes respectively.

ϕ°	20	25	30.75	35.25	40	45	50	55
α_{13}°	140	130	118.5	109.5	100	90	80	70
α_{12}°	83.3	79.7	74.85	70.5	65.6	60	54.1	47.9
α_{16}°	138	139.4	141.6	144	146	149	152	155
α_{15}°	59.5	66.3	74.88	82.0	89.9	98.4	107	116
ψ°	76.2	75.2	74.86	75.0	75.5	76.4	77.4	78.6
X°	44.2	37.2	30.7	26.6	22.9	19.4	16.5	14
h/ra	3.19	2.92	2.83	2.86	3.0	3.24	3.58	4.08

Table A1.4.2

These distributions of N=8 points provide the basis for global coverage gc(2) of the rotating Earth 24 hr per day where these N points are converted into N geostationary positions at latitudes +/- ϕ with adjacent longitudinal spacing $2\pi/N$ using Meyho Super-geosynchronous Circular Orbits.

A1.4.3 N=8 points points equally adjacently spaced $\pi/2$ in longitude at opposite latitudes +/- ϕ

The N=8 fixed points, $P_q=1,N$ may be located respectively above latitudes +/- ϕ , with $\pi/2$ longitudinal spacing between adjacent latitude points, yielding angular spacing between the points (Table A1.4.3),

$$\alpha_{12}=\alpha_{23}=\alpha_{34}=\alpha_{41}=\alpha_{56}=\alpha_{67}=\alpha_{78}=\alpha_{85}=\cos^{-1}[\sin^2(\phi)] \quad A1.4.3.1$$

$$\alpha_{15}=\alpha_{26}=\alpha_{37}=\alpha_{48}=2\phi \quad A1.4.3.2$$

$$\alpha_{16}=\alpha_{25}=\alpha_{27}=\alpha_{36}=\alpha_{38}=\alpha_{47}=\alpha_{45}=\alpha_{18}=\pi-\alpha_{12} \quad A1.4.3.3$$

$$\alpha_{17}=\alpha_{28}=\alpha_{35}=\alpha_{46}=\pi \quad A1.4.3.4$$

$$\alpha_{13}=\alpha_{24}=\alpha_{57}=\alpha_{68}=\pi-2\phi \quad A1.4.3.5$$

The minimum coverage angle ψ from any of the above points to provide gc(2) is derived from

$$\psi = \phi + \tan^{-1}[1/2\tan\phi] = \phi + X \quad A1.4.3.6$$

where the determining location is at latitude +/-X and the same 4 longitudes of the N points.

ϕ°	20	25	30	35.25	40	45	50	55
$\alpha 15^\circ$	40	50	60	70.5	80	90	100	110
$\alpha 13^\circ$	140	130	120	109.5	100	90	80	70
$\alpha 16^\circ$	96.7	100.3	104.5	109.5	114	120	126	132
$\alpha 12^\circ$	83.3	79.7	75.5	70.5	65.6	60	54.1	47.9
ψ°	73.9	72	70.9	70.5	70.8	71.6	72.8	74.3
X°	53.9	47	40.9	35.25	30.8	26.6	22.8	19.3
h/ra	2.61	2.24	2.06	2.0	2.04	2.17	2.38	2.69

Table A1.4.3

$\phi^\circ=35.25$ constitutes a uniform distribution of the N=8 points around a non-rotating sphere, equivalent to that in paragraph A1.4.1. (It constitutes a cube within a sphere).

These distributions of N=8 points provide the basis for global coverage gc(2) of the rotating Earth 24 hr per day where these N points are converted into N geostationary positions (loops) at latitudes $\pm\phi$ with adjacent longitudinal spacing $\pi/2$ using Meyho Super-geosynchronous Circular Orbits. Effectively, these N=8 points are either two distributions of N=4 points or four distributions of N=2 points with equal appropriate displacement. Although these configurations give better results in terms of minimum values of h/ra for the non-rotating sphere than those in paragraph A1.4.2, for global coverage gc(2) of the rotating Earth using Meyho Super-geosynchronous Circular Orbits they require networks with more satellites at greater heights above the surface of the rotating Earth.